16.b - Means and Variances

1) Look over Assignment 16.a
2) Reviewing Notes 16.b: Transformations and Combinations
3) Some more Dice Games
4) Being in a good place next Wednesday...

Pg. 383, #1, 2, 3, 4, 9, 11, 13, 6
16.a - Formulas/Calculation

**Measure of Center**

**Expected Value**

\[ \mu = E(X) = \sum x P(x) \]

**Measure of Spread**

**Variance**

\[ \sigma^2 = \text{Var}(X) = \sum (x - \mu)^2 P(x) \]

**Standard Deviation**

\[ \sigma = \text{SD}(X) = \sqrt{\text{Var}(X)} \]

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16.b - Rules about changing Means and Variances

1. Adding a constant to all values, adds a constant to the mean
   - \[ E(X \pm c) = E(X) \pm c \]
   - \[ \text{Var}(X \pm c) = \text{Var}(X) \]

2. Multiplying by a constant to all values, multiplies the mean
   - \[ E(aX) = a E(X) \]
   - \[ \text{Var}(aX) = a^2 \text{Var}(X) \]

3. Expected value of the sum is the sum of the expected values
   \[ E(X \leftrightarrow Y) = E(X) \leftrightarrow E(Y) \]
   (could be - too)

4. Variance of the sum is the sum of the variances
   \[ \text{Var}(X \leftrightarrow Y) = \text{Var}(X) \leftrightarrow \text{Var}(Y) \] (requires independence)
**Dice Game 1**

Consider a dice game: no points for rolling a 1, 2, 3; 5 points for a 4 or 5; 50 points for a 6. Find the expected value and the standard deviation.

<table>
<thead>
<tr>
<th>$X$ (points)</th>
<th>0</th>
<th>5</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>3/6</td>
<td>2/6</td>
<td>1/6</td>
</tr>
</tbody>
</table>

$E(X) = 0 \left( \frac{3}{6} \right) + 5 \left( \frac{2}{6} \right) + 50 \left( \frac{1}{6} \right)$

$E(X) = 0 + 10 + 50 \cdot \frac{1}{6}$

$E(X) = 10$

$V_{ar}(X) = (-10)^2 \left( \frac{3}{6} \right) + (-5)^2 \left( \frac{2}{6} \right) + (40)^2 \left( \frac{1}{6} \right)$

$V_{ar}(X) = 325$

**Dice Game 2**

Consider a dice game: no points for rolling a 1, 2, 3; 5 points for a 4 or 5; 50 points for a 6. Once you figure your point value, you double it.

Find the expected value and the standard deviation.

$E(2X) = 2E(X)$

$E(2X) = 2 \cdot 10$

$E(2X) = 20$

$V_{ar}(2X) = 2^2 V_{ar}(X)$

$V_{ar}(2X) = 4 \cdot 325$

$V_{ar}(2X) = 1300$

$E(B) = 20$

$SD(B) = 36.05$
**Dice Game 3**

Consider a dice game: no points for rolling a 1, 2, 3; 5 points for a 4 or 5; 50 points for a 6. You play the game for two rolls and add.

Find the expected value and the standard deviation:

\[
\begin{align*}
\mathbb{E}(X + X) &= \mathbb{E}(X) + \mathbb{E}(X) \\
&= 10 + 10 \\
&= 20
\end{align*}
\]

\[
\begin{align*}
\text{Var}(X + X) &= \text{Var}(X) + \text{Var}(X) \\
&= \frac{325}{650} + \frac{325}{650}
\end{align*}
\]

**Dice Game 4**

Consider a dice game: no points for rolling a 1, 2, 3; 5 points for a 4 or 5; 50 points for a 6. You roll and your friend rolls.

Find the expected value and the standard deviation of the difference between the two rolls:

\[
\begin{align*}
\mathbb{E}(X - X) &= \mathbb{E}(X) - \mathbb{E}(X) \\
&= 10 - 10 \\
&= 0
\end{align*}
\]

\[
\begin{align*}
\text{Var}(X - X) &= \text{Var}(X) + \text{Var}(X)
\end{align*}
\]
Assignment (Due Wednesday, January 20)

1) Pg. 383, #19-29 odd

2) Read Chapter 16, Pg. 372-377