11.3 - Continuous Compounding

*Some Worksheet Answers

*Idea of $e$ and where it comes from

*Continuous Compounding Interest Problems

*Assignment Time and Reading Time
11.3a: What is \( e \) ?

\( e \) is the "natural base"

\[
\approx 2.718281828...
\]

\( e \) is an irrational number (decimal above really does not have a pattern)

\( e \) is the limit as \( x \to \infty \) of \((1 + 1/x)^x\)

-look at results on calculator

11.3b: Continuous Compounding Interest

**Compound Interest**

\[
A = P \left( 1 + \frac{r}{n} \right)^{nt}
\]

- \( A \) = Balance After
- \( r \) = rate (annual)
- \( P \) = Principal
- \( t \) = time amount (years)
- \( n \) = compoundings per rate time (per year)

Let \( z = n/r \ldots \) and as \( n \) gets bigger...

\[
A = P \left( 1 + \frac{1/k}{z} \right)^{zt}
\]

\[
A = P \left( 1 + \frac{1/k}{z} \right)^{zt}
\]

\[
A = e^{rt}
\]

\[
8^{15} \quad (8^{3})^{5}
\]
11.3b : Continuous Compounding Interest

**Continuous Compounding Interest**

\[ A = Pe^{rt} \]

- \( A \) = Balance After
- \( r \) = rate (annual)
- \( P \) = Principal
- \( t \) = time amount (years)

A total of $15,000 is invested at an annual interest rate of 3% for ten years. Find the balance if the interest is compounded:

a) quarterly \[ n = 4 \]
\[ A = 15,000 \left( 1 + \frac{0.03}{4} \right)^{4 \times 10} \] = $20,240.30

b) monthly \[ n = 12 \]
\[ A = 15,000 (1.0025)^{40} \] = $20,247.64

c) daily \[ n = 365 \]
\[ A = 15,000 (1.3483486) \] = $20,247.64

d) Continuously
\[ A = Pe^{rt} \]
\[ A = 15,000e^{0.03 \times 10} \] = $20,247.88
A job pays a salary of $32,000 the first year. There is a 5.5% raise each year. What is the salary in the 40th year?

\[ 32,000 \left(1 + \frac{0.055}{1}\right)^{39} \]

Assignment (Due Tuesday, January 20)

1) Chapter 11 Problems Packet
   a) 11.1 / 11.2 / 11.63 / 11.64 / 11.69 / 11.70 / 11.53 / 11.54
   b) Fill in "continuous" for previous problems
   c) WS 11.65 / 11.66 / 11.67 / 11.68

*) Make sure that each problem is DONE
   (Detailed, Organized, Neat, Effort)

3) Read Chapter 11 pg. 148-151 (first half)