14.2 Rational Functions: Graph Properties

1) Quick Recap of Solving Systems

2) Rational Functions - Early Definition

3) Basic Graphing of Linear-to-Linear

4) Assignment Time

14.1 Assignment Problems

Solve the following systems by using GE to get into REF

\[
\begin{align*}
    a + b + c &= 6 \\
    2a - b + c &= -1 \\
    3a - c &= -7
\end{align*}
\]

\[
\begin{align*}
    2a + 2c &= 6 \\
    5a + 3b &= 11 \\
    3b - 4c &= 1
\end{align*}
\]

\[
\begin{align*}
    2a + b + 3c &= 1 \\
    2a + 6b + 8c &= 3 \\
    6a + 8b + 18c &= 5
\end{align*}
\]
14.2: Rational Functions

**Rational Function**

\[ f(x) = \frac{N(x)}{D(x)} \]

where \( N(x) \) and \( D(x) \) are polynomials with no common factors, and \( D(x) \) has at least a degree of one.

**Domain:** Remember that any domain values should not make the denominator of the function zero.

\[ f(x) = \frac{x^2 - 5}{x^2 - 3x^2 + 1} \]

For now, in this course, we will only look at rational functions that are called "linear-to-linear" which means the degree of both \( N(x) \) and \( D(x) \) are 1.

**Simple Rational Function Example**

Graph the function \( f(x) = \frac{1}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{1}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>undefined</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{1}{5} )</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1/2</td>
<td>2</td>
</tr>
<tr>
<td>( 1/3 )</td>
<td>3</td>
</tr>
<tr>
<td>( 1/5 )</td>
<td>5</td>
</tr>
<tr>
<td>-2</td>
<td>-0.5</td>
</tr>
<tr>
<td>-3</td>
<td>-0.33</td>
</tr>
<tr>
<td>-5</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

**Notice**

- the graph at each "end" approaches \( y = 0 \), but never gets there
- the graph at \( x = 0 \) does not exist
  - as \( x \) approaches 0 from the left, \( f(x) \) decreases without bound
  - as \( x \) approaches 0 from the right, \( f(x) \) increases without bound
14.2 : Rational Functions

Asymptotes
when the graphs y-values go off to infinities, the x-value where this happens (x = a) is a **vertical asymptote**

when the graphs y-values seem to "flatten", the y-value where this happens (y = b) is a **horizontal asymptote**

Vertial Asymptotes
occur at the zeros of D(x) “where the denominator is zero”

Horizontal Asymptotes
for linear-to-linear ....

y = coefficient of x in N(x)
coefficient of x in D(x)

Example 1
Graph the function

\[ f(x) = \frac{2x + 4}{x - 3} \]

**Vert Asym.** ⇒ \( x = 3 \) (D(x) = 0)

**Horiz Asym.** ⇒ \( y = \frac{2}{1} \) (leading coeff)

X-int \( ⇒ (0, -1\frac{3}{4}) \) (x = 0)

X-rnt \( ⇒ (-2, 0) \) (N(x) = 0)

\[ \begin{array}{c|c|c}
   x & y & \frac{2x + 4}{x - 3} \\
   \hline
   -4 & \frac{1}{4} & -5 \frac{1}{4} \\
   -3 & -1 \frac{3}{4} & -5 \frac{3}{4} \\
   2 & -3 \frac{1}{2} & -5 \frac{1}{2} \\
   2 \frac{1}{2} & -2 \frac{1}{2} & -6 \frac{1}{2} \\
   5 & -3 & -7 \frac{3}{4} \\
   5 \frac{1}{3} & 0 \frac{1}{3} & -8 \frac{1}{3} \\
\end{array} \]
Assignment ("Due" Thursday, February 25)

a) Packet 14 work
   14.1 (you do need to graph fairly accurate)

Read pg. 181-185
   (some real heavy fraction stuff!)