

1-7 Errors in Reasoning

Common errors in reasoning

1. Assume that the converse of a true statement is also true. In fact, the converse of a true statement may be true or it may be false.
2. Using an invalid statement to solve an equation.

Example 1 – “A parabola has a line of symmetry.”

- a. Is the statement True or False? If False, give a counterexample.

True, one side of a parabola is a reflection of the other side.

- b. Write the converse of the statement.

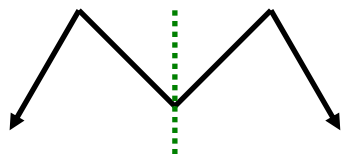
Write an if-then statement.

”If a graph is a parabola, then it has a line of symmetry.”

Now form the converse of this if-then statement.

”If a graph has a line of symmetry, then it is a parabola.”

- c. Is the converse True or False? If False, give a counterexample.
The converse is False. Not all graphs with a line of symmetry are parabolas.



Example 2 – Find and explain the error in this solution of the equation $3x^2 + 5x = 0$

Step 1 $3x^2 + 5x = 0$

Step 2 $3x^2 = -5x$

Step 3 $\frac{3x^2}{x} = \frac{-5x}{x}$

Step 4 $3x = -5$

$$x = \frac{-5}{3}$$

Error is in Step 3. Dividing both sides of the equation by x is a valid step only when $x \neq 0$. Notice that $x = 0$ is a solution to the original problem. The division in Step 3 has “lost” the solution $x = 0$.

The complete solution to the equation is $x = -5/3$ and $x = 0$.