# 2-5 Direct Variation with Powers (2 Days)

Direct variation

- *y* **varies directly** as *x* or
- *y* **is directly proportional** to *x*,
- if there exists a nonzero real number *k*, called the constant of variation, such that ***y = kx***

Direct variation with powers

- Let n be a positive real number.
- Then *y* **varies directly as the *n*th power of** *x* or
- *y* is **directly proportional to the *n*th power,** if there exists a nonzero real number *k* such that $y = kx^2$

## Solving Direct Variation Problems

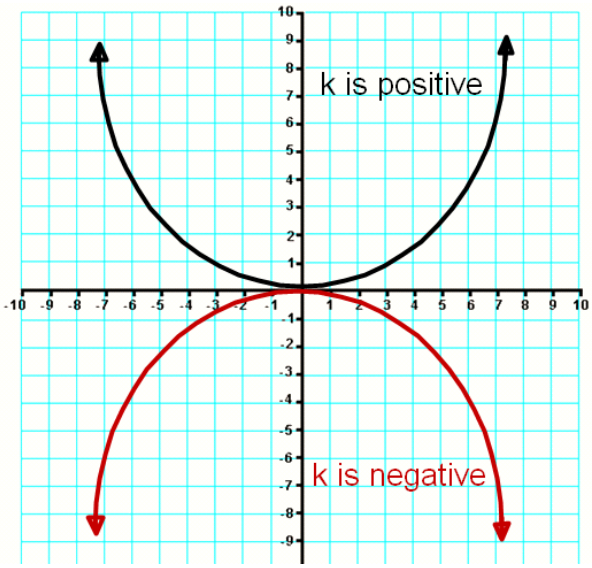
1. Write the general relationship among the variables as an equation.
   - Use the constant *k*
2. Substitute given values of the variables and find the value of *k*
3. Substitute this value of *k* into the equation from Step 1, obtaining a specific formula.
4. Substitute the remaining values and solve for the required unknown.

### Direct variation with the square

$y = kx^2$ k = variation constant $k \neq 0$

The graph is a curve called a parabola.

- with the vertex at (0,0)
- I and II quadrant when k is positive
- III and IV quadrant when k is negative




### The surface area of a sphere is direct variation with a square.

The dependent variable, surface area, varies directly with the square of the control variable, the radius.

$S.A. = 4\pi r^2$

Variation constant

**Example 1** - Find x when y = 10, $y = 2\pi x^2$

**Solution**

$10 = 2\pi x^2$ Plug in 10 for y

$10/2\pi = x^2$ Divide both sides by $2\pi$

$1.59 = x^2$

$\sqrt{1.59} = \sqrt{x^2}$

$x = \pm 1.26$

**Example 2** - Find y when x = 5, $y = 2\pi x^2$

**Solution**

$y = 2\pi x^2$ Plug in 10 for y

$y = 2\pi(5^2)$ Divide both sides by $2\pi$

$y = 50\ \pi$

$y = 157.1$

**Example 3** - Suppose y varies directly with the square of x, and y = 10 and x = 2.

a. What is the variation constant?
b. What is the value of y when x = 10?
c. What is the value of x when y = 40?

**Solution**

$y = kx^2$ "y" varies directly with the square of x

a. $10 = k\ (2)^2$
$10 = 4k$
$k = 2.5$

b. $y = 2.5x^2$
$y = 2.5(10)^2$
$y = 2.5(100)$
$y = 250$

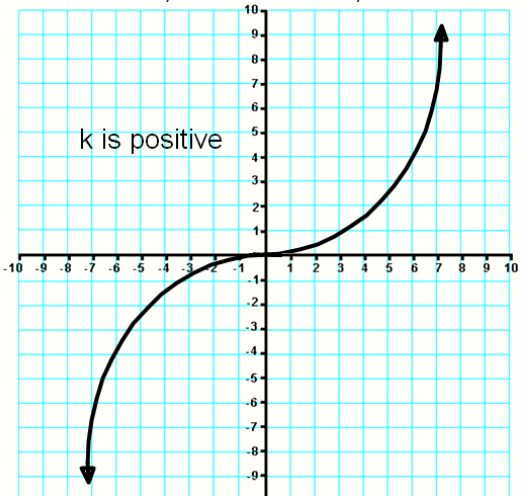
c. $40 = 2.5x^2$
$\sqrt{16} = \sqrt{x^2}$
$x = 4$ Since length is positive, only find the positive square root.

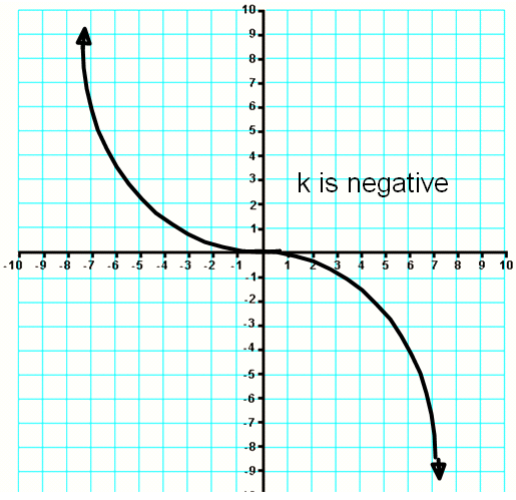
**Direct variation with the cube**

$y = kx^3$ k = variation constant $k \neq 0$

The graph is a curve through the origin.

- I and III quadrant when k is positive
- II and IV quadrant when k is negative
- y varies directly with the cube of x, $x^3$



**Volume of a sphere is direct variation with a cube.**

$$V = \frac{4}{3}\pi r^3$$

**Example 3** - Find y when x = 3, $y = 1.2x^3$

Solution

$y = 1.2x^3$

$y = 1.2\ (3)^3$

$y = 1.2\ (27)$

$y = 32.4$

**Example 4** - Find x when y = 915, $y = 1.2x^3$

Solution

$915 = 1.2x^3$

$762.5 = x^3$

$x = 9.1$

Note – provided with 2 out of 3 and solve for the third. You need to know what formula to use.

## Homework Day 1

- Read pg. 91-95
- Practice 13 1-3, 6-8
- Pg. 96 #1, 4-7, 15-18, 32-34

## Day 2 - Direct Variation with Powers - Story problems

**Example 1** - The height reached by a pole-vaulter varies directly with the square of the speed of the vaulter at the point to pole reaches the ground.

Pole-vaulter reaches 16 ft at 32 ft/sec. How high will a vaulter reach with speed of 35 ft./sec.

Solution

$h = ks^2 \quad 16 = k32^2 \quad 16 = 1024k$

$\frac{16}{1024} = k \quad k = .015625$

$h = .015625 \bullet 35^2 \quad h = 19.1 ft.$

**Example 2** - The leap height of a basketball player varies directly with the square of the "hang time." When the leap height is in feet and the hang time is in seconds, the variation constant is 4.

If the hang time of a basketball player is about 0.73 seconds, what is the leap height?

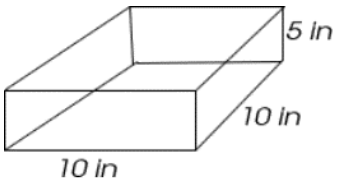
Solution

Let L = leap height
$L = 4t^2 \sim 4(0.73)^2 \sim 2.13$ ft

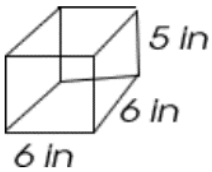
**Example 3** - The boxes shown all have square bases and the same height, 5 in.

Write an equation that models the relationship between the volume, V, of each box, and the length, x, of an edge of each base. What kind of variation is this model?

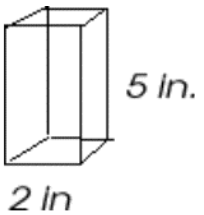
$y = kx^3$ k = variation constant $k \neq 0$




Volume: 500 in$^3$




Volume: 180 in$^3$




Volume: 20 in$^3$

**Solution**

The formula for the volume of a rectangular prism is V = base • height.
For these boxes, let the area of the base be $x^2$, and the height is 5.
The model equation is $V = x^2 \cdot 5$ or $V = 5x^2$
This is direct variation with a square.

**Example 4 -** Use the equation to find the length of the edges of the base of a box whose volume is 96.8 in 3.

Solution

$V = 5x^2$

$96.8 = 5x^2$ (Substitute 96.8 for V)

$19.36 = x^2$ (Divide both sides by 5)

$4.4 = x$ (Since the length must be positive, find the positive square root.)

The length of the base edge is 4.4 in.

**Example 5 -** How does the volume change when the length of the base edge is tripled?

**Solution**

To see what happens when the length of the base edge is tripled, substitute 3x for x in the variation equation.

$V = 5(3x)^2$

$V = 5(9x)^2$

$V = 45x^2$

Use a ratio to compare this result to the original equation.

$$\frac{\text{volume of box with base edge } 3x}{\text{volume of box with base edge } x} = \frac{45x^2}{5x^2} = \frac{9}{1}$$

If the base edge is triples, then the volume is multiplied by 9.

**Example 6**

The number of watts of power generated by a windmill varies directly with the cube of the wind speed in miles per hour. The variation constant is 0.015.

Suppose a wind is blowing at 35 mph. Find the number of watts of power generated by a windmill in this wind.

Solution

$$w = 0.015s^3$$
$$= 0.015(35)^3$$
$$= 643.13$$

The number of watts of power generated is about 643.

## Homework Day 2

- Pg. 96 #8-14, 19-24
- Practice 13 #9-13
- Pg. 650 Skill 21 (Slope)