2-6 Using Powers – Fractional Exponents
Day 1

Warm-up
Solve

1. Find the surface area and volume of a sphere with radius r = 4cm.

   Solution

   $SA = 4\pi r^2$

   $SA = 4\pi 4^2$

   $SA = 4\ \pi 16$

   $SA = 64\pi$

   $\boxed{SA = 201.06\ cm^2}$

   $V = \frac{4}{3}\pi r^3$

   $V = \frac{4}{3}\pi 4^3$

   $V = \frac{256}{3}\pi$

   $\boxed{V = 268.08\ cm^3}$

2. Evaluate and round to the nearest tenth: x^5 when x = 1.3.

   Solution

   x^5 when $x = 1.3$

   $1.3^5 = 3.7$

Negative Exponents
Complete the table

Negative Exponent	Decimal	Fraction
2^{-1}		$\frac{1}{2}$
2^{-2}		$\frac{1}{4}$
2^{-3}		$\frac{1}{8}$
2^{-4}		$\frac{1}{16}$
2^{-5}		$\frac{1}{32}$

Using the information from the table, draw a conclusion about negative exponents.

Negative Exponent Rule

$$\boxed{x^{-n} = \frac{1}{x^n} \text{ and } \frac{1}{x^{-n}} = x^n, \text{ when } x \neq 0.}$$

Example 1 – Simplify and write answers with positive exponents.

a. $6y^{-4}$ Solution: $6y^{-4} = \frac{6}{y^4}$

b. $4x^{-2}$ Solution: $4x^{-2} = \frac{4}{x^2}$

c. $\frac{3y^{-2}}{6x^{-2}}$ Solution: $\frac{3y^{-2}}{6x^{-2}} = \frac{x^2}{2y^2}$

Zero Exponent

Complete the table

1^0	
2^0	
3^0	
4^0	
5^0	

What can we say about x^0?

Zero Exponent Rule

$x^0 = 1$, when $x \neq 0$

Example

a. $3^0 = 1$

b. $1{,}000^0 = 1$

Example 2 – Simplify and write answers with positive exponents.

a. $\frac{4x^{-2}}{y^0}$ Solution: $\frac{4x^{-2}}{y^0} = \frac{4}{x^2}$

b. $\frac{2m^{-2}}{n^4}$ Solution: $\frac{2m^{-2}}{n^4} = \frac{2}{m^2 n^4}$

c. $5^2 x^0$ Solution: $5^2 x^0 = 25$

d. $\frac{6^0}{(2y)^{-2}}$ Solution: $\frac{6^0}{(2y)^{-2}} = 1 \bullet (2y)^2 = 2^2 y^2 = 4y$

e. $(3n)^{-2}$ Solution: $(3n)^{-2} = \frac{1}{(3n)^2} = \frac{1}{3^2 n^2} = \frac{1}{9n^2}$

f. $\frac{2^0 4^{-3}}{x^2}$ Solution: $\frac{2^0 4^{-3}}{x^2} = \frac{1}{4^3 x^2} = \frac{1}{64x^2}$

When the exponent is positive, leave it alone!
When the exponent is negative, move it and make it positive!

Homework

- Read pg. 99 - 102
- Pg. 102 #1-10, 39-42
- Pg. 642 Skill 11 evens
- Pg. 642 Skill 12 odds

Day 2

Warm-up

Simplify

1. $(5m^{-6})(10n^2)$

   Solution: $(5m^{-6})(10n^2) = \frac{5 \bullet 10n^2}{m^6} = \frac{50n^2}{m^6}$

2. $\frac{12s^{-6}}{2t^{-4}}$

   Solution: $\frac{12s^{-6}}{2t^{-4}} = \frac{6t^4}{s^6}$

Fractional Exponents and Radical Form

Find

a. $25^{\frac{1}{2}}$ Solution: $25^{.5} = 25^{\frac{1}{2}} = \sqrt{25} = 5$

b. $36^{\frac{1}{2}}$ Solution: $36^{.5} = 36^{\frac{1}{2}} = \sqrt{36} = 6$

c. $27^{\frac{1}{3}}$ Solution: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Therefore, $x^{\frac{1}{2}} = \sqrt{x}$, when x ≥ 0 and $x^{\frac{1}{3}} = \sqrt[3]{x}$.

$x^{\frac{1}{2}} = \sqrt{x}$, when $x \geq 0$.

$x^{\frac{1}{3}} = \sqrt[3]{x}$

Exponential form Radical form

Example 1 - Rewrite in radical form

a. $81^{\frac{1}{2}}$

b. $125^{\frac{1}{3}}$

c. $(-8)^{\frac{1}{3}}$

d. $5w^{\frac{1}{2}}$

e. $(5w)^{\frac{1}{2}}$

f. $3z^{\frac{1}{3}}$

Example 2 – Rewrite in exponential form

a. $5\sqrt{x}$

b. $\sqrt{5x}$

c. $\sqrt[3]{y}$

d. $-7\sqrt{25}$

Example 3 – Find the value of each expression when x = 8 and y = 9.

a. $3x^{-3}y^{\frac{1}{2}}$

b. $\dfrac{5^0 x^{\frac{1}{3}}}{(2y)^{-2}}$

Example 4 – Suppose a roller coaster car can travel at a speed of 30 ft/s. What would be the greatest radius of a vertical loop of a track it could make?

Solution

Use the formula $V = (32r)^{\frac{1}{2}}$.

$V = (32r)^{\frac{1}{2}}$

$30 = (32r)^{\frac{1}{2}}$

$30 = \sqrt{32r}$

$(30)^2 = (\sqrt{32r})^2$

$900 = 32r$

$\dfrac{900}{32} = \dfrac{32r}{32}$

$r = 28.125$

The radius of the track would be about 28 ft.

Example 5 – The diameter of a cylinder with volume V and height h is given by the expression $d = 2\left[V(\pi h)^{-1}\right]^{\frac{1}{2}}$.

Find the diameter, to the nearest tenth, of a cylinder with volume 200 cm^3 and height 8 cm.

Solution

$d = 2\left[V(\pi h)^{-1}\right]^{\frac{1}{2}}$

$V = 200$

$h = 8$

Plug the values for V and h into the expression.

$d = 2\left[200(\pi 8)^{-1}\right]^{\frac{1}{2}}$

$d = 2\left[200(\frac{1}{\pi 8})\right]^{\frac{1}{2}}$

$d = 2\left[\frac{200}{8\pi}\right]^{\frac{1}{2}}$

$d = 2\sqrt{\frac{200}{8\pi}}$

$d = 2\frac{\sqrt{200}}{\sqrt{8\pi}} = 2 \bullet 10\frac{\sqrt{2}}{\sqrt{8\pi}} = 20\frac{\sqrt{2}}{\sqrt{8\pi}} = \frac{20}{2}\frac{\sqrt{2}}{\sqrt{2\pi}} = 10\frac{1}{\sqrt{\pi}} = 5.64$

The diameter is about 5.6 cm.

Review – Write an equation to represent each situation. Use *k* to represent the variation constant.

a. The amount of clay needed to make a square pyramid with height 1 ft. varies directly with the square of the length of a base edge.
   Solution
   $V = ke^2$, where V = amount of clay and e = the base edge.

b. The time it takes to fill a spherical balloon is directly proportional to the cube of the radius of the balloon.
   Solution
   $t = kr^3$, where t = time and r = the radius of the balloon.

Homework

- Read pg. 99 - 102
- Pg. 102 #12-29, 32, 34, 36
- Practice 14 #9-26