2-7 Doubling and Halving (2 Days)

Day 1

Warm-up

Write with positive exponents.

1. $(-3c^{-1})(d^{0})$
2. $\dfrac{16k^{0}n^{2}m^{-3}}{4m^{-2}}$
3. What is the value of $12^{\frac{1}{2}}$ to the nearest tenth.
4. Rewrite using fractional exponents $-4\sqrt[3]{x}$
5. Rewrite in radical form $(4mn)^{\frac{1}{2}}$
6. If you have \$25 and you double it five times, how much do you have now?
   \$25 • 2 = \$50 • 2 = \$100 • 2 = \$200 • 2 = \$400 • 2 = \$800
7. Find the value of 2^7. 2 • 2 • 2 • 2 • 2 • 2 • 2 = 128
8. What is the value of \$100 • 2^0? \$100 • 1 = \$100
9. What is one-half of 2^{10}? 2^{10} = 1024/2 - 512
10. A savings account containing \$1500 doubles every three years. How much money will be in the account after 12 years? Year 3 \$3000, year 6 \$6000, year 9 \$12,000, year 12 \$24,000. Had 4 doubling periods.

If I take a sheet of paper and tear it in ½, how many pieces will I have? If I tear it again, how many pieces will I have? The number of pieces will double each time. This is called a doubling period.

Doubling Period – The amount of time it takes a quantity to double.

Example – Volume of a landfill doubles every 3 years. The doubling period is 3 years.

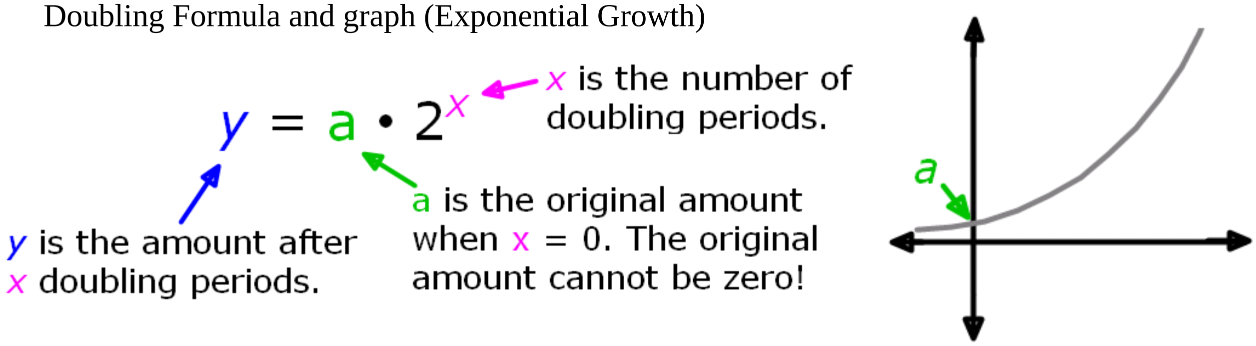
This is **exponential growth** – The increase of a quantity at the same rate at regular intervals.

Exponential Functions

- Base is a constant
- Power is a variable

Exponential functions with base 2 – Used when something doubles in quantity.

Doubling Formula and graph (Exponential Growth)



Example 1 – Solve $y = 5 \cdot 2^x$, when x = 15

Solution

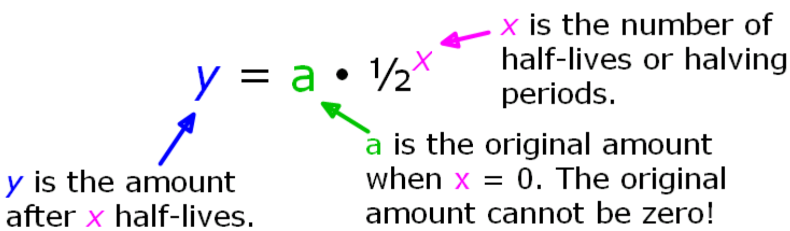
$y = 5 \cdot 2^{15}$ Plug in 15 for x.

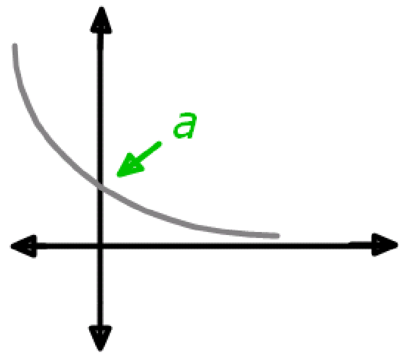
$y = 5 \cdot 32{,}768$

$\boxed{y = 163{,}840}$

Exponential functions with base ½ – Used when something is halved.

Exponential Decay formula and graph





Example 2 – Solve $y = 80 \cdot \frac{1}{2}^x$, when x = 4

Solution

$y = 80 \cdot \frac{1}{2}^4$ Plug in 4 for x, the number of half-life periods

$y = 80 \cdot .0625$

$\boxed{y = 5}$

Homework

- Read pg. 105 – 109
- Practice 15 #1-12
- Pg. 110 #3-8, 22-32

Day 2

Warm-up

1. If the value of a \$1000 investment doubles every 15 years, find the value of the investment 75 years after it was made?
2. A type of bacteria reproduces by dividing into two bacteria every 6 min. If you start with one bacterium, how many bacteria will there be after one hour?
3. After 2 hours?

Example 3 – Using the equation $5 \cdot 2^x$, how many doubling periods when y = 50,000?

Solution

Solve for x

$y = 5 \cdot 2^x$ Write the equation from example 1

$50{,}000 = 5 \cdot 2^x$ Plug 50,000 in for y

$10{,}000 = 2^x$ Divide each side of the equation by 5

Guess and check to find x.

$2^{10} = 1024$	To low
$2^{11} = 2048$	To low
$2^{12} = 4096$	To low
$2^{13} = 8192$	To low
$2^{14} = 16,384$	To high

The value of x is between 13 and 14. Continue to guess and check.

$2^{13.5} = 11583.24$	To high
$2^{13.3} = 10,085.54$	To high
$2^{13.2} = 9,410.17$	To low

$x = 13.3$ The closest value to 10,000

Another approach to solving

Solve for x (number of doubling periods) ...plug in 50,000 for y

$$50000 = 5 \cdot 2^x$$

Use guess and check to find the value of x

Values of x	values of y		
10	$(y = 5 \cdot 2^{10})$	= 5,120	*too low so try >10
13	$(y = 5 \cdot 2^{13})$	= 40,960	*close but lets try >13
14	$(y = 5 \cdot 2^{14})$	= 81,1920	*too high-try b/t 13§14
13.3	$(y = 5 \cdot 2^{13.3})$	=50,427.7	*closest!!!

So $x = 13.3$ *so there are 13.3 doubling periods

Example 4 – Suppose Joe invests $2500 in a mutual fund at age 25. If the value of his investment doubles every seven years, what will be the value of the fund when Joe is 60 years old?

Solution

First find the number of doubling periods

$60 - 25 = 35$	Number of years of investing
$35/7 = 5$	Doubles every 7 years. So, there are 5 doubling periods

Let v = value of the fund

$V = a \cdot 2^x$	write the doubling formula
$V = 2500 \cdot 2^5$	Plug in 2500 for a, the initial amount and 5 for x, the number of doubling periods.
$V = 2500 \cdot 32$	
$V = 80,000$	

The value of the fund when Joe is 60 years old will be $80,000.

Example 5 - You have $500 invested in an account that doubles your money every 6 years.

Write a function that models the amount of money in the account.

Solution

$y = 500 \cdot 2^x$

How much will be in the account 10 years from now?

Solution

$y = 500 \cdot 2^x$

Find the x, the number of doubling periods

Doubles every 6 years. Find the number of doubling periods in 10 years.

10/6 = number of doubling periods in 10 years.

$y = 500 \cdot 2^{10/6}$

$y = 500 \cdot 3.17$

$y = 1{,}587.40$

The account will be worth $1,587.40 in 10 years.

About how long will it take to have $10,000?

Solution

$y = 500 \cdot 2^x$

$10{,}000 = 500 \cdot 2^{x/6}$ Plug 10,000 in for y. Need to find the number of years. Keeping in mind the doubling is every 6 years so, the doubling period is x/6. Solve for x.

$20 = 2^{x/6}$ Divide each side of the equation by 500.

Guess and check to find x, the numbers of years.

$2^4 = 16$

$2^5 = 32$

So, x/6 is between 4 and 5.

$x/6 = 4$ $x/6 = 5$

$x = 24$ $x = 30$

x is between 24 and 30.

Continue to guess and check to come closest to $20 = 2^{x/6}$

$x = 25$, $2^{x/6} = 17.96$

$x = 26$, $2^{x/6} = 20.16$

About 26 years to get $10,000.

Another approach to the solution

Use the equation $y = a \cdot 2^x$ because this is exponential growth

Plug in 10,000 for y and 500 for a…

$$10{,}000 = 500 \cdot 2^x$$

Use guess and check to solve for x…

Doubling time	amount	
6 years	$ 1000	* 500*2 = 1000
12	$ 2000	
18	$ 4000	
24	$ 8000	
30	$ 16000	*so 10,000 is somewhere b/t

24&30 yrs

Try 26 years:

26years/6 = 4.33 so plug in 4.33 into $y = 500 \cdot 2^{4.33}$

$y = 500(20.1)$

$y = 10{,}056.10$

So it takes 26 years to have $10,000.

Example 6 - A medication has a half-life in the bloodstream of 2 hours. The dosage of the medicine is 3mg. How much is left after 5 hours?

Solution

This is exponential decay so use the halving formula: $y = a(1/2)^x$

First, find the number of halving periods (x)

5 hours/2 hours = 2.5

Plug in 2.5 for x and 3mg for a…

$y = 3(1/2)^{2.5}$

$y=3(0.17)$

$y = 0.53$

Example 7 – Half-life is 20 days. Initial amount is 10g.

1. Write an equation that models the exponential decay.

   Solution

   $a = 10$

   $y = 10 \cdot 1/2^x$

2. After 30 days, how much is left?

   Solution

   Find the number of half-lives.

   30/20 = 1.5 — 1.5 halving periods

   $y = 10 \cdot 1/2^{1.5}$ — Plug 1.5 in for x, the number of halving periods

   $y = 10 \cdot .35$

   $y = 3.5$

   3.5g are left after 30 days.

3. In how many days will you have 2 grams?

   Solution

   $y = 2$

   $2 = 10 \cdot 1/2^x$ — Plug 2 in for y, the ending amount

   Guess and check to solve for x

   $x = 1, y = 5$

   $x = 2, y = 2.5$

   $x = 3, y = 1.25$

   $x = 2.5, y = 1.77$

   $x = 2.3, y = 2.03$

   About 2.3 halving periods.

   $2.3 \cdot 20 = 46$ days

Another approach - In how many days will you have 2 grams left?

Solution

This is exponential decay so use the halving formula; $y = a(1/2)^x$

Plug in 2 for y (amount after x half-lives) and 10 for a (original amount)

$$2 = 10(1/2)^x$$

Use guess and check to find the value of x that will make the equation equal 2…

Time	Amount
0 days	10 g
20 days	5 g
40 days	2.5 g
60 days	1.25 g *between 40 & 60 days

Try 45 days:

45 days/ 20 days = 2.25 plug in 2.25 for x

$y = 10(1/2)^{2.25}$

$y = 2.1$ g *lets see if we can get closer to 2 g

Try 46 days:

46 days/ 20 days = 2.3 plug in 2.25 for x

$y = 10(1/2)^{2.3}$

$y = 2.03$ g

so it will take **46 days** for there to be only 2 g left

Example 8 – Using the formula $y = 80 \cdot \frac{1}{2}^x$, how many halving periods will it take to get y = 1?

Solution

$y = 80 \cdot \frac{1}{2}^x$ Equation from example 5

$1 = 80 \cdot \frac{1}{2}^x$ Plug 1 in for y and solve for x.

Guess and check to find x

x	y
5	2.5
6	1.25
7	.625
6.1	1.16629
6.2	1.08818
6.3	**1.01532**
6.4	.9473

About 6.4 halving periods.

Homework

- Practice 15 #13-15
- Pg. 110 #9-11, 16-19