Section 3-2 Solving Systems by Substitution

Warm-up:

1. What is the value of y in the equation 2x + 3y = 9 when 5 is substituted for x?
   Solution: $y = -1/3$
2. Solve the equation x + y = 6 for x.
   Solution: $x = 6 - y$
3. Replace y in the equation x – 3y = 10 with x + 4. What is the new equation?
   Solution: $x - 3(x + 4) = 10$
4. Solve the equation x – 3(x + 4) = 10.
   Solution:
   $$x - 3x - 12 = 10$$
   $$-2x - 12 = 10$$
   $$+12 \quad +12$$
   $$\underline{-2x} = \underline{22}$$
   $$-2 \quad -2$$
   $$\boxed{x = -11}$$
5. If x = -11, find the value of y in the equation x – 3y = 10.
   Solution:
   $$x - 3y = 10$$
   $$-11 - 3y = 10$$
   $$+11 \quad +11$$
   $$\underline{-3y} = \underline{21}$$
   $$-3 \quad -3$$
   $$\boxed{y = -7}$$

3-2 Solving Systems by Substitution

Methods for solving systems:

1. Graphing - less precise, have to estimate the solution.
2. Substitution
3. Elimination

We solved systems of equations by graphing and had to estimate the solutions. Graphing is less precise.

The substitution method provides an exact solution to the system of equations.

The substation method is based on the ides that at the place where two lines cross, both x-coordinates and both y-coordinates must be the same.

Therefore, the point of intersection is the solution to the system of equations.

You can use substitution to transform a system of two equations in two variables into one equation with one variable.

It doesn’t matter which equation or which variable you pick. There is no right or wrong choice. The answer will be the same regardless.

But, some choices are better than others.

The four steps to solving a system of equations by substitution.

1. Solve one of the equations for one of the variables.
2. Use the result from step 1 to substitute for the appropriate variable in the other equation and solve the resulting equation.
3. Substitute the value found in step 2 into the equation from step 1.
4. Check the results of step 2 and step 3 in both of the original equations.

Example 1: Solve the system of equations by substitution.

$y = 7x - 1$

$3y + 3 = x$

Solution

$y = 7x - 1$ ⟵ Equation 1

$3y + 3 = x$ ⟵ Equation 2

Step 1: The first equation has a variable already isolated. Already solved for y.

Step 2: Substitute 7x-1 for y in the second equation.

$3y + 3 = x$

$3(7x-1) + 3 = x$ ⟵ Substitute 7x-1 for y and sovle for x

$21x - 3 + 3 = x$

$21x = x$

$-x \quad -x$

$20x = 0$

$x = 0$

Step 3: Find the value for y when x=0.

$y = 7x - 1$ ⟵ Plug 0 in for x and solve for y. Can use either equation.

$y = 7(0) - 1$

$y = -1$

The solution (x, y) is (0, -1).

Step 4: Check the solution in both of the original equations.

$y = 7x - 1$ ⟵ Equation 1

$-1 = 7(0) - 1$

$-1 = -1$ ⟵ Equation 1 is true with (0, -1)

$3y + 3 = x$ ⟵ Equation 2

$3(-1) + 3 = 0$

$-3 + 3 = 0$

$0 = 0$ ⟵ Equation 2 is true with (0, -1)

Therefore, the ordered pair (0, -1) is the solution for the system of equations.

The solution is always written as an ordered pair because it is the point of intersection of the two equations.

Example 2: Solve this system of equations by substitution.

$x + 2y = 1$

$5x - 4y = -23$

Solution:

$x + 2y = 1$ ⟵ Equation 1

$5x - 4y = -23$ ⟵ Equation 2

Step 1: Solve one equation so a variable is isolated. The 1st equation is the easiest to solve. Solve the first equation for x.

$x + 2y = 1$ ⟵ Solve for x

$-2y \quad -2y$

$x = -2y + 1$

Step 2: Substitute $-2y+1$ for x in the second equation.

$5(-2y + 1) - 4y = -23$ ⟵ Substitute $-2y+1$ for x

Solve the new equation for y.

$5(-2y + 1) - 4y = -23$

$-10y + 5 - 4y = -23$ ⟵ Distributive property and combine like terms.

$-14y + 5 = -23$

$-5 \quad -5$

$-14y = -28$

$$\frac{-14y}{-14} = \frac{-28}{-14}$$

$y = 2$

Now we need to find x.

Step 3: Find the value for x when y=2. You can use either of the original equations.

$x + 2y = 1$ ⟵Use the equation from step 1 to find the value of x

$x + 2(2) = 1$ ⟵Plug 2 in for y and solve for x

$x + 4 = 1$

-4 -4

$x = -3$

The solution (x, y) is (-3, 2).

Step 4: Check the solution in both of the original equations.

$x + 2y = 1$ ⟵Equation 1

-3+4 = 1 ⟵Plug in the ordered pair (-3, 2)

1= 1 ⟵Equation 1 is true with (-3, 2)

$5x - 4y = -23$ ⟵Equation 2

5(-3)-4(2)=-23 ⟵Plug in the ordered pair (-3, 2)

-15-8=-23

-23 = -23 ⟵Equation 2 is true with (-3, 2)

Therefore, the ordered pair (-3, 2) is the solution for the system of equations.

Example 3: Solve by substitution

$r = 3q$

$r = -0.4q + 1.7$

Solution

$r = 3q$ ⟵Equation 1

$r = -0.4q + 1.7$ ⟵Equation 2

Step 1: Both equations are already solved for r.

Step 2: Substitute the value of r from equation 1 into equation 2 and solve for q.

$r = -0.4q + 1.7$

$3q = -0.4q + 1.7$ ⟵Substitute 3q for r in equation 2

+0.4q +0.4q

3.4q=1.7

$\frac{3.4q}{3.4} = \frac{1.7}{3.4}$

$q = .5$

Step 3: Substitute $q=.5$ into the equation in step 1. Can use either of the original equations.

$r = 3q$

$r = 3(.5)$ ⟵ Plug in .5 for q and solve for r

$r=1.5$

The solution (q, r) is (.5, 1.5).

Step 4: Check the results in both the original equations.

$r = 3q$ ⟵ Equation 1

$1.5 = 3(.5)$ ⟵ Plug in the ordered pair (.5, 1.5)

$1.5 = 1.5$ ⟵ Equation 1 is true (.5, 1.5)

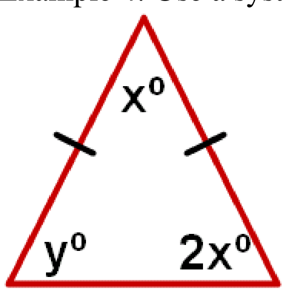
$r = -0.4q + 1.7$ ⟵ Equation 2

$1.5 = -0.4(.5) + 1.7$ ⟵ Plug in the ordered pair (.5, 1.5)

$1.5 = -.2 + 1.7$

$1.5 = 1.5$ ⟵ Equation 2 is true (.5, 1.5)

Example 4: Use a system of equations to find the measures of the angles of the triangles.



Hint: Because the sides are the same length, the angles opposite these sides are also equal in measure.

Solution

Step 1: Use known facts about triangles to write two equations.

1. The sum of the measures of the angles in a triangle is 180°.
   $x+2x+y=180$
2. The measures of the base angles of an isosceles triangle are equal.
   $y=2x$

Step 2: Solve the system of equations by substitution.

$x+2x+y=180$

$y=2x$

The second equation is already solved for y.

Substitute the value of y (=2x) from the second equation into the first equation and solve for x.

$x+2x+y=180$

$x+2x+2x=180$ ⟵Substitute the value for y from equation 2 into equation 1 and solve for x.

$5x=180$

$\frac{5x}{5}=\frac{180}{5}$

$x=36$

Step 3: Find the value of y.
$y = 2x$
$y=2(36)$
$y=72$

The solution (x, y) is (36, 72)

The measures of the angles are 36°, 72°, and 72°.

Example 5: A total of 1096 people attended the concert at the State Fair. Reserved seats cost $25 and unreserved seats cost $20. If $26,170 was collected, how many of each type of ticket was sold?

Hint: Use the information to write 2 equations. One equation about number of people (quantity) and the second equation about dollar amount.

Solution
Step 1: Write a system of equations. Solve one equation for one variable.
A table may help organize the data
Let r = reserved seats
Let u = unreserved seats

	Number of tickets sold	Dollar amount taken in
Reserved Seats	r	25r
Unreserved Seats	u	20u
Total	1096	26,170

$r + u = 1096$ Number of tickets sold
$25r + 20u = 21{,}170$ Dollar amount collected

Solve one of the equations so you have one variable that is isolated.
Easiest to solve the first equation for r
$r = -u + 1096$

Step 2: Solve the system of equations by substitution. Substitute the value of r from the first equation into r in the second equation. Solve for u.

$25r + 20u = 26,170$

$25(-u + 1096) + 20u = 26,170$

$-25u + 27,400 + 20u = 26,170$

$-5u + 27,400 = 26,170$

$\quad -27,400 \quad -27,400$

$-5u = -1230$

$$\frac{-5u}{-5} = \frac{-1230}{-5}$$

$u = 246$

Step 3: Plug u=246 into the first equation and solve for r.

$r + 246 = 1096$

$\quad -246 \quad -246$

$r = 850$

The solution (r, u) of the system is (850, 246).

There were 850 reserved and 246 unreserved seats sold.

Example 6: Solve by substitution.

$7x + 2y = 16$

$-21x - 6y = 24$

Solution

Step 1: Solve one of the equations for a variable.

$7x + 2y = 16$

$-21x - 6y = 24$

Solve the first equation for y

$7x + 2y = 16$

$-7x \quad -7x$

$2y = -7x + 16$

$$\frac{2y}{2} = \frac{-7x + 16}{2}$$

$$y = \frac{-7}{2}(x) + 8$$

Step 2: Substitute the value of y in step 1 into the second equation.

$$-21x - 6y = 24$$
$$-21x - 6\left[\left(-\frac{7}{2}\right)x + 8\right] = 24$$
$$-21x + 21x - 48 = 24$$
$$-48 = 24$$

The answer is nonsense, -48 cannot equal 24. What does the answer mean? Remember, a solution is where the lines intersect. These lines are parallel and do not intersect. Therefore, no solution.

$$-21x - 6y = 24$$
$$-6y = 21x + 24$$
$$y = \frac{-21}{6}x - 4$$
$$y = \frac{-7}{2}x - 4$$

Solving the second equation for y, you see that the equations have the same slope. The lines are parallel and will never intersect.

Homework:
Read pg. 129 - 132
Pg. 132 #2, 4-7, 8b, 10-14, 17, 18